SCHOOL: ___SOLUTION___

TEAM: _____

RELAY: ___1___

1. $1/36$
2. 4
3. -2
4. 220

SCHOOL: ___SOLUTION___

TEAM: _____

RELAY: ___2___

1. 4
2. 400
3. 7305
4. $5/2\pi$

SCHOOL: ___SOLUTION___

TEAM: _____

RELAY: ___3___

1. 3
2. $40/3$
3. $\sqrt{3}$
4. -1

SCHOOL: _____

TEAM: _____

RELAY: _____

1. _____
2. _____
3. _____
4. _____

Seventh Annual Upper Peninsula
High School Math Challenge

Northern Michigan University (Marquette MI, USA)

Saturday 12 March 2016

RELAY 1   Category: PROBABILITY   PLAYER 1

I roll three dice. What is the probability that the sum of the three numbers showing is 5?

Pass your answer to PLAYER 2.

$$
\left.\begin{matrix}
113 \\
131 \\
311 \\
122 \\
212 \\
221
\end{matrix}\right\} \quad \frac{6 \text{ ways}}{6^3 = 216 \text{ total}} \qquad \frac{6}{216} = \frac{1}{36}
$$

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RELAY 1　　　　　Category: FACTORING　　　　　PLAYER 2

Express the number you receive from PLAYER 2 as $\frac{p}{q}$, a reduced fraction. Compute the value of pq. How many of pq's positive integer factors are perfect squares?

Pass your answer to PLAYER 3.

$$\frac{P}{q} = \frac{1}{36} \qquad pq = 36 = 2^2 \cdot 3^2$$

Factors are $1, 2^2, 3^2, 6^2 : 4$ total

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RELAY 1 Category: QUADRATIC EQUATIONS PLAYER 3

Let k be the number you receive from PLAYER 2. The quadratic equation $x^2 + kx + r = 0$ has one (repeated) root. What is the value of that root?

Pass your answer to PLAYER 4.

$k = 4$

$x^2 + 4x + r = 0$

sum of roots is $\frac{-b}{a} = -4$

repeated root must be -2 (and -2)

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RELAY 1   Category: POLYGONS   PLAYER 4

Let k be the number you receive from PLAYER 3. A polygon has $k + 8$ sides. All the angles but one have a measure of $100°$. What is the measure, in degrees, of the remaining angle?

Run your answer to the front.

$k = -2$ Polygon has 6 sides.

Total angle is $(6-2) \cdot 180 = 720$

Five ~~have~~ are $100°$.

$720 - 5(100) = 220$

Seventh Annual Upper Peninsula High School Math Challenge

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Saturday 12 March 2016

RELAY 2   Category: CIRCULAR EQUATIONS   PLAYER 1

A circle is given by the equation $x^2 - 2x + y^2 - 4y = 11$. What is the radius of this circle?

Pass your answer to PLAYER 2.

$$x^2 - 2x \qquad + y^2 - 4y \quad = 11$$

$$x^2 - 2x + 1 \qquad + y^2 - 4y + 4 = 11 + 1 + 4$$

$$(x-1)^2 + (y-2)^2 = 16 = 4^2$$

Center: $(1, 2)$

radius: 4

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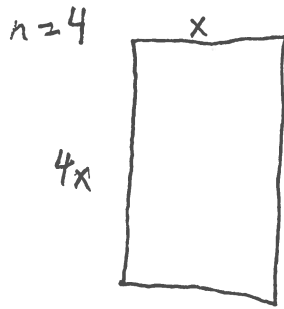
Saturday 12 March 2016

RELAY 2   Category: AREA AND PERIMETER   PLAYER 2

Let n be the number you receive from PLAYER 1.

A rectangle is n times as long as it is wide. If the perimeter of the rectangle is 100 cm, what is the area of the rectangle in square centimeters?

Pass your answer to PLAYER 3.

$n = 4$

x

$4x$

$4x + x = 50$

$5x = 50$

$x = 10$

Area: $40 \cdot 10 = 400$

Seventh Annual Upper Peninsula
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Saturday 12 March 2016

RELAY 2   Category: GREGORIAN CALENDAR   PLAYER 3

Let n be the positive square root of the number you receive from PLAYER 2.

How many days are in the n-year period ending with the year 2016?

Pass your answer to PLAYER 4.

$$n = \sqrt{400} = 20$$

In 20 years, there are 5 leap years,

$$20(365) + 5 = 7305$$

Seventh Annual Upper Peninsula
High School Math Challenge

Northern Michigan University (Marquette MI, USA)

Saturday 12 March 2016

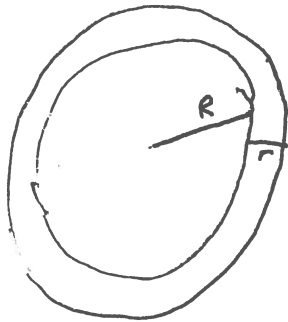RELAY 2   Category: CIRCULAR GEOMETRY   PLAYER 4

Let n be the remainder when the number you receive from PLAYER 3 is divided by 100.

A circle with a circumference of 10 miles is centered within a second circle with circumference n inches longer than the first. What is the distance between the edges of these circles, in inches?

Run your answer to the front.

$$n = 7305 \bmod 100 = 5$$

Inner Circumference: $2\pi R$

Outer Circumference: $2\pi(R+r) = 2\pi R + n$

$$2\pi(R+r) = 2\pi R + n$$

$$2\pi R + 2\pi r = 2\pi R + n$$

$$2\pi r = n$$

$$r = \frac{n}{2\pi} = \frac{5}{2\pi}$$

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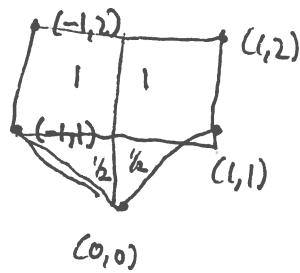
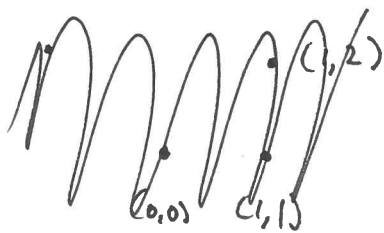Saturday 12 March 2016

RELAY 3   Category: AREA   PLAYER 1

A pentagon has vertices (in order) at $(0,0)$, $(1,1)$, $(1,2)$, $(-1,2)$ and $(-1,1)$. What is the area of this pentagon?

Pass your answer to PLAYER 2.

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RELAY 3   Category: WORK PROBLEM   PLAYER 2

The number you receive from PLAYER 1 is the number of pizzas I can eat in an hour. If you can eat half that number in an hour, how many minutes will it take for both of us to share one pizza?

Pass your answer to PLAYER 3.

I eat 3 pizzas in an hour

You eat $\frac{3}{2}$ pizzas in an hour

Together we eat $\frac{9}{2}$ pizzas in an hour

So it takes $\frac{2}{9}$ hours to eat a pizza

$\frac{2}{9}$ hr $\cdot \frac{60 \text{ min}}{\text{hr}}$ = $\frac{2}{3} \cdot 20$ min = $\frac{40}{3}$ min

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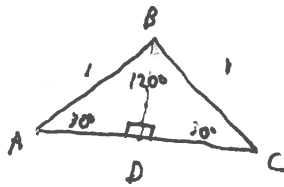
RELAY 3   Category: ISOSCELES TRIANGLES   PLAYER 3

Express the number you receive from PLAYER 2 as a reduced fraction $\frac{p}{q}$.

One angle of an isosceles triangle measures pq degrees. If the two sides of the triangle have length 1, what is the length of the third side?

Pass your answer to PLAYER 4.

$$\frac{p}{q} = \frac{40}{3} \qquad pq = 120$$

$\angle ABD$ is $30-60-90$

$BD = \frac{1}{2}$

$AD = \sqrt{1^2 - (\frac{1}{2})^2} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$

$AC = 2AD = \sqrt{3}$

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RELAY 3   Category: CUBIC EQUATIONS   PLAYER 4

A cubic equation $x^3 + bx^2 + cx + d = 0$ (b, c, d all integers) has three real roots. One of these roots is 1, another of the roots is the number you receive from PLAYER 3. What is the value of b?

Run your answer to the front.

one root is 1

one root is $\sqrt{3}$

third root is conjugate of $\sqrt{3}$ since coefficients are integers

Sum of roots $= -\frac{b}{9} = -b$

$1 + \sqrt{3} + (-\sqrt{3}) = -b$

$b = -1$